Properties of Logarithms

A Logarithm is an Exponent: It is the exponent we put on some base to get *x*. $x > 0$

Common Logarithm Base 10	Natural Logarithm Base e	General Logarithm Base b
$\log x = y$ $10^y = x$	$\ln x = y$ $e^y = x$	$\log_b x = y$ $b^y = x$
$\log 1 = 0$	$\ln 1 = 0$	$\log_b 1 = 0$
$\log 10 = 1$	$\ln e = 1$	$\log_b b = 1$
$\log 10^x = x$ for all x	$\ln e^x = x$ for all x	$\log_b b^x = x$ for all x
$10^{\log x} = x \quad x > 0$	$e^{\ln x} = x \quad x > 0$	$b^{\log x} = x \quad x > 0$
Product Property $\log AB = \log A + \log B$ and $\log A + \log B = \log AB$	Product Property $\ln AB = \ln A + \ln B$ and $\ln A + \ln B = \ln AB$	Product Property $\log_b AB = \log_b A + \log_b B$ and $\log_b A + \log_b B = \log_b AB$
Quotient Property $\log \frac{A}{B} = \log A - \log B$ and $\log A - \log B = \log \frac{A}{B}$	Quotient Property $\ln \frac{A}{B} = \ln A - \ln B$ and $\ln A - \ln B = \ln \frac{A}{B}$	Quotient Property $\log_b \frac{A}{B} = \log_b A - \log_b B$ and $\log_b A - \log_b B = \log_b \frac{A}{B}$
Power Property $\log B^t = t \log B$ and $t \log B = \log B^t$	Power Property $\ln B^t = t \ln B$ and $t \ln B = \ln B^t$	Power Property $\log_b B^t = t \log_b B$ and $t \log_b B = \log_b B^t$

Some **misconceptions:**

1) $\log(a+b) = \log a + \log b$ **NOT TRUE** $\log(a+b) \neq \log a + \log b$

$\log(a-b) \neq \log a - \log b$ But what does "$\log(a+b)$" or "$\log(a-b)$" mean?

The LOG IS AN EXPONENT. Therefore, $\log(a+b)$ must be the exponent we put on 10 to get $(a+b)$. i.e. $y = \log(a+b)$ means $10^y = a+b$. Similarly, $y = \log(a-b)$ means $10^y = a-b$

2) $\log(ab) \neq (\log a)(\log b)$ and $\log\left(\frac{a}{b}\right) \neq \frac{\log a}{\log b}$ and $\log\left(\frac{1}{a}\right) \neq \frac{1}{\log a}$

3) $\log ab^t \neq t \log ab$ Use product property: $\log ab^t = \log a + \log b^t$

but $\log(ab)^t$ does $= t\log(ab)$