Properties of Logarithms

A Logarithm is an Exponent: It is the exponent we put on some base to get x. x > 0

Common Logarithm Base 10	Natural Logarithm Base e	General Logarithm Base b
$ \begin{aligned} \log x &= y \\ 10^y &= x \end{aligned} $	$ \ln x = y \\ e^y = x $	$\log_{\mathbf{b}} x = y$ $\mathbf{b}^{y} = x$
log 1 = 0	In 1 = 0	log _b 1 = 0
log 10 = 1	In e = 1	log b = 1
$\log 10^x = x \text{for all } x$	$\ln e^x = x$ for all x	$\log_b b^x = x$ for all x
$10^{\log x} = x \qquad x > 0$	$e^{\ln x} = x \qquad x > 0$	$b^{\log x} = x \qquad x > 0$
Product Property log AB = log A + log B and log A + log B = log AB	Product Property In AB = In A + In B and In A + In B = In AB	Product Property log AB = log A + log B and log A + log B = log AB
$\frac{\text{Quotient Property}}{\log \frac{\mathbf{A}}{\mathbf{B}} = \log \mathbf{A} - \log \mathbf{B}}$ and $\log \mathbf{A} - \log \mathbf{B} = \log \frac{\mathbf{A}}{\mathbf{B}}$	Quotient Property $\ln \frac{A}{B} = \ln A - \ln B$ and $\ln A - \ln B = \ln \frac{A}{B}$	$\frac{\text{Quotient Property}}{\log_b \frac{\mathbf{A}}{\mathbf{B}} = \log_b \mathbf{A} - \log_b \mathbf{B}}$ and $\log_b \mathbf{A} - \log_b \mathbf{B} = \log_b \frac{\mathbf{A}}{\mathbf{B}}$
Power Property log B' = t log B and t log B = log B'	Power Property $ \ln B' = t \ln B $ and $ t \ln B = \ln B' $	Power Property $\log_b B' = t \log_b B$ and $t \log_b B = \log_b B'$

Some misconceptions:

log (a + b) = log a + log b NOT TRUE log (a + b) ≠ log a + log b log (a - b) ≠ log a - log b But what does "log (a + b)" or "log (a - b)" mean?

The LOG IS AN EXPONENT. Therefore, $\log (a + b)$ must be the exponent we put on 10 to get (a + b). i.e. $y = \log (a + b)$ means $10^y = a + b$. Similarly, $y = \log (a - b)$ means $10^y = a - b$

- 2) $\log (ab) \neq (\log a)(\log b)$ and $\log \left(\frac{a}{b}\right) \neq \frac{\log a}{\log b}$ and $\log \left(\frac{1}{a}\right) \neq \frac{1}{\log a}$
- 3) log ab^t ≠ t log ab Use product property: log ab^t = log a + log b^t but log(ab)^t does = t log(ab)