Worksheet 11: Eigenvalues and eigenvectors

Example 0.74. Let

$$A = \begin{bmatrix} 5 & -1 \\ & 0 \end{bmatrix}, v_1 = \begin{bmatrix} 1 \\ \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

Compute Av_i for i = 1,2,3. Are they multiples of v_i?

Example 0.75. Let

$$A = \begin{bmatrix} 1 & -2 \\ -1 & \end{bmatrix}, v = \begin{bmatrix} 2 \\ \end{bmatrix}.$$

Determine if v is an eigenvector of A. If yes, find the corresponding eigenvalue.

Example 0.76. Determine if -4 is an eigenvalue of the matrix $A = \begin{bmatrix} 1 & 6 \\ & 2 \end{bmatrix}$. If yes, find all eigenvectors associated to it.

Example 0.77. It is known that the following matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

has two eigenvalues, 1 and 3. Find a basis for each of the two eigenspaces corresponding to them. What are the geometric multiplicities of the eigenvalues?

Example 0.78. The matrix

$$\mathbf{A} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

is not invertible because 0 is an eigenvalue

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0 \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Example 0.79. For each of the following matrices $\mathbf{A}$, first find an expression in λ for $\det(\mathbf{A} - \lambda\mathbf{I})$:

(a) $\mathbf{A} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

(b) $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$

and then use it to find the eigenvalues of $\mathbf{A}$ and corresponding algebraic multiplicities.

Example 0.80. Find the eigenvalues of

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 2 \end{bmatrix}$$

This is an example of a matrix that has complex eigenvalues.

Example 0.81. Determine the eigenvalues of the following matrix

$$\mathbf{A} = \begin{bmatrix} 1 & & \\ & 2 & \\ & & 3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & & \\ 4 & 2 & \\ 5 & 6 & 3 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 4 & 5 \\ & 2 & 6 \\ & & 3 \end{bmatrix}.$$